Grade 6 Mathematics, Quarter 2, Unit 2.1

Understand the Concepts of Ratios and Unit Rates to Solve Real-World Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand the concept of a ratio.
- Use ratio language to describe ratio relationships.
- Understand the concept of unit rate.
- Use ratio and rate reasoning to solve real-world and mathematical problems.
- Solve unit rate problems involving unit price.
- Solve unit rate problems involving constant speed.

Mathematical practices to be integrated

Attend to precision.

- Communicate effectively about ratio and unit rates.

Look for and make use of structure.

- Express ratios and rates in multiple forms.

Essential questions

- What is a ratio?
- How do you use ratio language to describe ratio relationships?
- What is the difference between a ratio and a rate?
- What is a unit rate?
- How can you use what you learn about ratios to solve real-world math problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

6. RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6. RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several

objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In previous grades, students have been exposed to parts of a whole and parts of parts when working with fractions.

Current Learning

Students understand the concept of a ratio as a way of expressing relationships between quantities. They know when a ratio is describing part-to-part or part-to-whole comparisons. Students understand that a rate is a relationship between two quantities with different units of measure. Students understand that when using rates such as "a:b," "b" cannot equal zero. Unit rates in sixth grade are limited to non-complex fractions.

Future Learning

In seventh grade students will compute unit rates associated with ratios of fractions, including ratios of length, area, and other quantities measured in like or different units. Students will also recognize and represent proportional relationships between two quantities. They will use proportional relationships to solve multistep ratio and percent problems.

Additional Findings

According to *Curriculum Focal Points,* "students use simple reasoning about multiplication and division to solve ratio and rate problems." (p. 18)

According to *A Research Companion to Principles and Standards for School Mathematics*, "how students understand a concept has important implications for what they subsequently can do and learn . . . for example, ratio language versus fraction language." (p. 95)

Grade 6 Mathematics, Quarter 2, Unit 2.2

Applications of Ratios and Rates

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Use multiple forms—including tables, tape diagrams, double number line diagrams, or equations—to show equivalent ratios.
- Use the concept of a percent being part of a whole at per-100 rate.
- Convert measurement units appropriately by multiplying or dividing quantities.
- Make tables relating quantities, whole-number measurements, and missing values to compare ratios.
- Plot on a coordinate plane to compare ratios.

Mathematical practices to be integrated

Attend to precision.

- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.

Look for and make use of structure.

- Look closely to discern patterns in tables relating quantities, whole-number measurements, missing values, and structures of ratios.
- Manipulate and transform units appropriately when multiplying and dividing quantities.

Essential questions

- How do you convert a rate to a unit rate?
- How do you compare ratios on a coordinate plane?
- What does percent compare?
- How can you use a table to compare ratios?
- Why is it important to compare measures using the same type of unit?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Common Core State Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In previous grades, students have been exposed to parts of a whole and parts of parts when working with fractions. In fifth grade, they applied previous understanding of multiplication and division to the operation of dividing the numerator by the denominator. Students have worked with fraction models and equations to represent real-world problems.

Current Learning

Students understand the concept of a ratio as a way of expressing relationships between quantities. They know when a ratio is describing part-to-part or part-to-whole comparisons and can create and interpret tables accordingly. Students understand the conversion of measurement units when multiplying and dividing quantities. Students can find percent of a quantity as a rate per 100. Students can plot pairs of values on the coordinate plane. This is developmental learning.

Future Learning

In seventh grade, students will compute unit rates associated with ratios of fractions, including ratios of length, area, and other quantities measured in like or different units. Students will also recognize and represent proportional relationships between two quantities. They will further use proportional relationships to solve multistep ratio and percent problems.

Additional Findings

According to *Curriculum Focal Points*, "students expand their repertoire of problems that they can solve by using multiplication and division, and they build their understanding of fractions to understand ratios." (p. 18)

According to *A Research Companion to Principles and Standards for School Mathematics,* "the fact that to understand objects as entailing a single or multiple proportion relies on understanding measures as ratio comparisons highlights the special way in which multiplication must be conceived." (p. 104)

According to *Common Core Progressions,6-7: Ratio and Proportional Reasoning*, "ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life." (p. 2)

Grade 6 Mathematics, Quarter 2, Unit 2.3

Identifying and Generating Equivalent Expressions Through Properties to Solve Real-World Problems

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Generate equivalent expressions using the properties of operations.
- Recognize when two expressions are equivalent.
- Reason about problems by writing and solving equations with nonnegative rational numbers.
- Reason about and solve real-world equations with one variable.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships to generate equivalent expressions.

Attend to precision.

- Communicate to others how two expressions are equivalent.

Essential questions

- How do you use the properties of operations to generate equivalent expressions?
- How is an equation like a balance scale?
- What steps do you take to solve equations involving variables?
- What strategies can be used to solve for unknowns?
- How do you show that expressions are equivalent?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

6. EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

6. EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6. EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In fourth grade, students learned that numbers written in different forms are equal. Students used the distributive property with numbers only. Students wrote and interpreted numerical expressions without variables.

Current Learning

In sixth grade students apply and extend previous understanding of the properties of operations to generate equivalent expressions. They identify equivalent expressions and write and solve real-world mathematical problems for nonnegative rational numbers. Students solve one-variable equations and inequalities. This is a critical area and needs to reach the reinforcement stage of development.

Future Learning

Students will use properties of operations to generate equivalent expressions. They will solve real-world and mathematical problems using numerical and algebraic expressions and equations.

Additional Findings

According to *Curriculum Focal Points*, "students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems [and] they solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation." (p. 18)